Study of Higher Order Dispersion Effects on Soliton Interaction In Dispersion Shifted Fiber

Endra[1], S. El Yumin[2]
Department of Opto-Electrotechniques And Laser Applications, University of Indonesia, Jakarta
[1]Department of Computer Engineering, University of Bina Nusantara, Jakarta
[2] Department of Electrical Engineering, ISTN, Jakarta

Abstract— **We studied higher order dispersion (β_3) effects on soliton interaction in dispersion shifted fiber (DSF) with solving numerically the Nonlinear Schrödinger Equation (NLSE). Soliton interaction in the fiber determines the bit rate transmission distance product of communication systems. We used a soliton pair that has secant hyperbolic profile as input to the NLSE to analyze soliton interaction. Initial soliton power given a fundamental soliton and fiber loss is negligible. We used Symmetrized Split Step Fourier (SSSF) method to solve the NLSE . We compare soliton interaction in DSF from result of NLSE with β_3 is included and β_3 is negligible. And we find optimum value of parameter soliton pair so can obtain the maximum bit rate transmission distance product.**

Keywords— **Higher order dispersion (β_3), soliton interaction, dispersion shifted fiber (DSF), Nonlinear Schrödinger Equation (NLSE), Symmetrized Split Step Fourier (SSSF) method.**

I. INTRODUCTION

The word *soliton* was coined by Krusal and Zabusky in 1965 to describe the particle like properties of pulse envelope in dispersive nonlinear media; under certain condition the envelope not only propagates undistorted but survives collisions just as particles do [1].

In 1973 Hasegawa and Tappert suggested the possibility of soliton propagation in optical fiber in their paper on *Applied Physic. Letter. 23, 142 (1973)*. Soliton in optical fibers were first observed by L.F. Mollenauer *et al*. in an expriments that used a mode-locked color-center laser to obtain short ($T_{FWHM} \approx 7$ ps, *FWHM = Full Width at Half Maximum*) optical pulse near 1.55 μm, a wavelength close to which the fiber loss is minimum [2].

DSF is fiber that zero dispersion wavelength shifted to near 1,55 μm, here fiber loss is minimum. In DSF when operating wavelength is 1.55 μm, GVD (Group Velocity Delay) parameter (β_2) is close to zero so higher order dispersion (β_3) must be considered [1].

The propagation of optical pulse envelope for pulse width ≥ 0.1ps in a single mode, lossless fiber, included higher order dispersion and maintain its polarization is goverend by Nonlinear Shrődinger Equation (NLSE), can be written as [1]

$$\frac{\partial A}{\partial z}+\frac{i}{2}b_2\frac{\partial^2 A}{\partial T^2}-\frac{1}{6}b_3\frac{\partial^3 A}{\partial T^3}=ig\,|A|^2A \qquad (1)$$

where :

A = Pulse envelope (slowly varying variable).
g = Non-linearity parameter.

$T = t - z/vg$, where T = time in a reference frame moving with pulse; t = physical time, z = the direction of pulse propagation along fiber axis and v_g = the group velocity at the center wavelength.

Eq (1) can be written in normalized form as

$$\frac{\partial U}{\partial x}+\mathrm{sgn}(b_2)\frac{i}{2}\frac{\partial^2 u}{\partial t^2}-\mathrm{sgn}(b_3)\frac{b_3}{6}\frac{\mathrm{L_D}}{\mathrm{L_D'}}\frac{\partial^3 u}{\partial t^3}=iN^2|U|^2U \qquad (2)$$

$$L_D=\frac{T_0^2}{|b_2|};\, L_D'=\frac{T_0^3}{|b_3|};\, L_N=\frac{1}{g\,P_0};\, x=\frac{z}{L_D};\, N^2=\frac{L_D}{L_N}$$

$U=\frac{A}{\sqrt{P_0}}$; sgn(β_2) (or sgn(β_3)) takes values +1 and -1 depending on whether β_2 (or β_3) is positive or negative [1].

U, P_0, ξ, T_0, L_D, L_D' and L_N is normalized pulse envelope, peak input pulse power, input pulse width, normalized distance, dispersion length, higher order dispersion length and nonlinear length.

Soliton exist only for integer values of N, soliton corresponding to $N = 1$ is called fundamental soliton [1].

II. METHODOLOGY

We use SSSF method to solve numerically equation (2), algorithm of SSSF method is written using MatLab.

A. *Symmetrized Split Step Fourier (SSSF) method*

Eq (2) can be written as [2]

$$\frac{\partial U}{\partial x}=\left(\hat{D}+\hat{N}\right)U \qquad (3)$$

$\hat{D}$ and $\hat{N}$ is linear operator and nonlinear operator, where

$\hat{D}=-\mathrm{sgn}(b_2)\frac{i}{2}\frac{\partial^2 u}{\partial t^2}+\mathrm{sgn}(b_3)\frac{1}{6}b_3\frac{\mathrm{L_D}}{\mathrm{L_D'}}\frac{\partial^3 u}{\partial t^3}$;

$\hat{N}=iN^2|U|^2$.

SSSF method nonlinear effects assumed at the middle of step size segment and calculated its effect for the whole segment [3]. The approximation solution of NLSE by SSSF method is

$$U(x+h,T)=\exp\left(\frac{h}{2}\hat{D}\right)\exp\left(\int_{x}^{x+h}\hat{N}(x')dx'\right)\exp\left(\frac{h}{2}\hat{D}\right)U(x,T) \quad (4)$$

$$\int_{x}^{x+h}\hat{N}(x')dx' \approx \frac{h}{2}\left(\hat{N}(x)+\hat{N}(x+h)\right)$$

h is normalized step size parameter. Accuration of SSSF method is third order of h ($O(h^3)$). h chosen so that the maximum phase shift (Φ_{max}) due the nonlinear operator is below certain value as

$$g\left|A_p\right|^2 h \le \frac{\Phi_{\max}}{L_D} \quad (5)$$

A_p is peak pulse envelope. It has been reported that when Φ_{max} is below 0.05 rad, SSSF method gives a good result for simulation of most contemporary optical communication systems [4].

B. Soliton Interaction

We simulated soltion interaction with a pair soliton as input to the NLSE as [2]

$$U(0,\tau)=\sec h(\tau-q_0)+r\sec h\left(r(\tau+q_0)\right)e^{i\theta} \quad (6)$$

Where r is relative amplitude of two solitons, θ is relative phase of pair and $2q_0$ (normalized to T_0) is the initial separation. q_0 is related to the bit rate as [2]

$$B=\frac{1}{2q_0T_0} \quad (7)$$

For r = 1, θ = 0 and β_3 = 0 , a pair of soliton along the fiber is showing periodic collapse due the mutual interaction where the separation q(ξ) varies periodically along the fiber with the oscillation period as [2]

$$\xi_p=\frac{\pi\sinh(2q_0)\cosh(q_0)}{2q_0+\sinh(2q0)} \quad (8)$$

Unnormalized oscillation period (z_p) is

$$z_p=x_pL_D \quad (9)$$

z_p limited the maksimum transmission distance (L_T) with design criterion as [2]

$$L_T \ll z_p \quad (10)$$

Using Eq (7), Eq (8) , Eq (9) and defenition of L_D, bit rate transmission distance product written as

$$B^2L_T \ll \frac{\xi_p}{4q_0^2\left|b_2\right|} \quad (11)$$

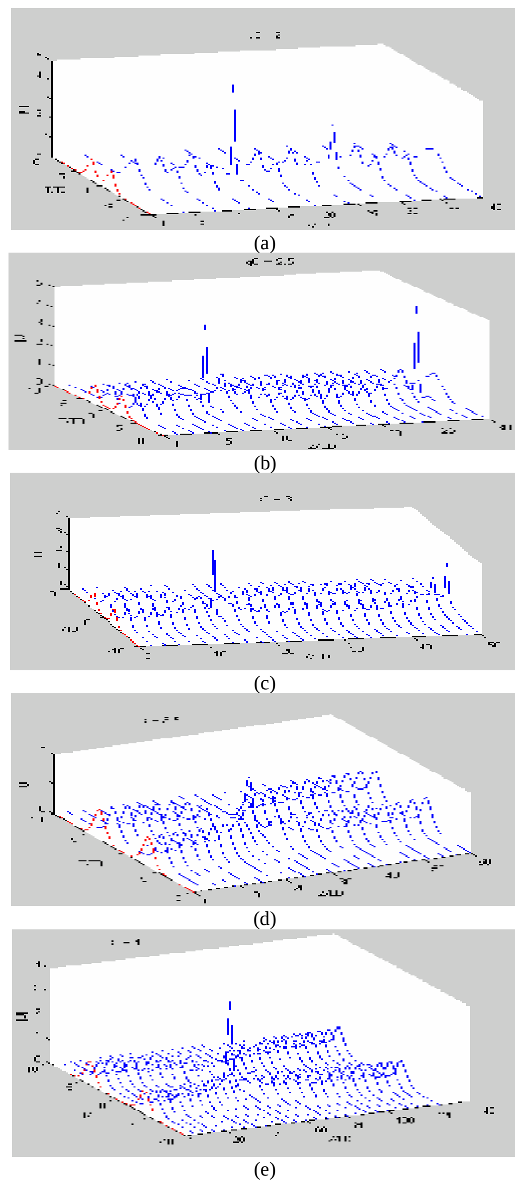
We use Eq (6) as input to the Eq (2) then compare to the Eq (8) to analyze how β_3 effects on the soliton interaction. We also change parameter r to get maximum ξ_p so that bit rate transmission distance product can made optimum.

III. Results

A. Higher Order Dispersion Effects on Soliton Interaction

Eq (2) solved numerically by SSSF method with parameters that we used are λ_0 = 1.55 µm, T_0 = 10 ps, β_2 = -1 ps^2/km, β_3 = 0.1 ps^3/km, γ = 0.003 /mW.km, N = 1, and h = 0.001. A soliton pair as in Eq (6) with parameter r = 1, and θ = 0 as input to Eq (2).

The result of SSSF mehod to solve Eq (2) plot in Fig (1) for q_0 = 2, 2.5, 3, 3.5, and 4. The value of oscillation period (ξ_p) with comparison to Eq (8) in Table I and plot of ξ_p as a function of q_0 in Fig (2) .



(a) (b) (c) (d) (e)

Fig. 1. Evolution of a soliton pair along the fiber, r =1, θ = 0 (a) q_0 = 2 (b) q_0 = 2.5 (c) q_0 = 3 (d) q_0 =3.5 (e) q_0 = 4.

TABLE I
COMPARISON OF OSCILLATION PERIOD

q_0	ξ_p By SSSF Method β_3 is included	ξ_p By Eq (8) β_3 is negligible
2	4	10.308
2.5	7.5	18.049
3	14	30.715
3.5	24	51.409
4	55	85.333

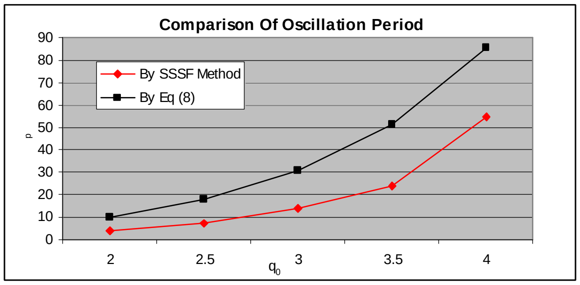


Fig. 2 Plot of ξ_p as a function of q_0.

B. *Effects of relative ampiltude on soliton interaction*

We use the same parameter as before except parameter r as variable. A soliton pair as in Eq (6) with parameter $q_0 = 2$, and $\theta = 0$ as an input to Eq (2). The result of SSSF mehod to solve Eq (2) for r = 0.9, 1, and 1.1 plot in Fig (3). The value of oscillation period (ξ_p) is in Table II.

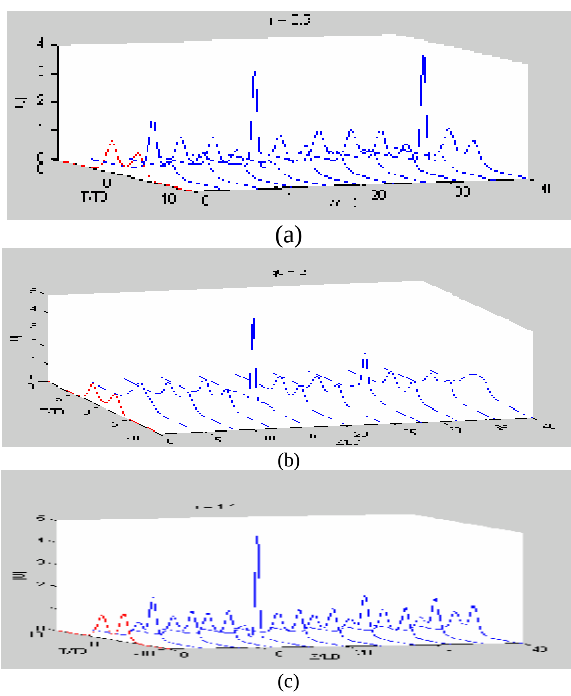


Fig. 3. Evolution of a soliton pair along the fiber, $q_0 = 2$, $\theta = 0$ (a) $r = 0.9$ (b) $r = 1$ (c) $r = 1.1$.

TABLE II
OSCILLATION PERIOD FOR DIFFERENT *r*

r	ξ_p
0.9	12
1	4
1.1	12

IV. Discussion

From Fig (1), we can see a pair of soliton in DSF also undergo periodic collapse due the mutual interaction where the separation, $q(\xi)$, varies periodically along the fiber. We also can see in Table I that β_3 effects reduces the value of ξ_p. For $q_0 = 2, 2{,}5, 3, 3{,}5$ and 4, the reduction of ξ_p is *61.2 %, 58.5 %, 54.4 %, 53.32 %* and *35.5 %* from ξ_p which the higher order dispersion effects is ignored. When q_0 increase, the reduction of ξ_p by β_3 become smaller, so for large q_0 , the effects of β_3 can be ignored. Fig (2) is showing comparison plot of ξ_p in DSF as a function of q_0 between β_3 negligible and included.

From Table II, we can see that relative amplitude, r, of a pair soliton effect on ξ_p, where both for r = 0.9 and 1.1 can increase ξ_p three times.

Table III is showing comparison of bit rate transmisson distance product in DSF using Eq (11) and Table I between β_3 is included and β_3 is negligible, also for conventional fiber, with β_2 = -20 ps^2/km and β_3 = 0.

Table IV is showing comparison bit rate transmisson distance product for different r in DSF using Eq (11) and Table II.

TABLE III
COMPARISON OF BIT RATE
TRANSMISSION DISTANCE PROCUCT

q_0	B^2L_T ((Tb/s)^2.Km) in DSF β_3 is included	B^2L_T ((Tb/s)^2.Km) in DSF β_3 is negligible	B^2L_T ((Tb/s)^2.Km) in conventional fiber
2	<< 0.25	<< 0.644	<< 0.03
2.5	<< 0.30	<< 0.722	<< 0.036
3	<< 0.389	<< 0.853	<< 0.043
3.5	<< 0.490	<< 1.049	<< 0.052
4	<< 0.859	<< 1.333	<< 0.067

TABLE IV
BIT RATE TRANSMISSION DISTANCE PRODUCT
FOR DIFFERENT *r*

r	B^2L_T ((Tb/s)^2.Km)
0.9	<< 0.75
1	<< 0.25
1.1	<< 0.75

From Table III, we can see in DSF β_3 effects reduces the value of ξ_p and the bit rate transmission distance product. Although β_3 effects in DSF reduces the value of ξ_p but have still larger bit rate transmisson distance product than conventional fiber.

From Table IV, we can see that relative amplitude, r, of a pair soliton effect on bit rate transmisson distance product, where both for r = 0.9 and 1.1 is increasing three times than for r = 1.

V. Conclusion

In DSF when β_3 is included, a pair of soliton also experienced periodic collapse due the mutual interaction where the separation varies periodically along the fiber. β_3 reduces the value of ξ_p, but for large q_0 , the effects of β_3 can be ignored. The reducing of ξ_p by β_3 effects in DSF reduces the bit rate transmission distance product, but still larger than conventional fiber.

Relative amplitude of a pair soliton effects on ξ_p, for appropriate r less differ from 1, ξ_p is increasing and increases the bit rate transmisson distance product.

Acknowledgment

Endra Author thanks to ADB and University of Bina Nusantara for My scholarship, to My lab's network administrator, Mr Ronald Suwandi, for the computer that I used and to Yuliana for her support and motivation.

References

[1] Agrawal, P. Govind, *Fiber-Optic Communication Systems,* Willey Series In Microwave And Optical Engineering, (1992).

[2] Agrawal, P. Govind, *Nonlinear Fiber Optics*, Academic Press, Sandiego, LA, (1989).

[3] Natee Wongsangpaiboon, *Variational Calculation of Optimum Dispersion Compensation For Non-Linear Dispersive Fibers*, Thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University, Blacksburg, Virginia (May 17, 2000).

[4] Jong-Hyung Lee, *Analysis and Characterization of Fiber Nonlinearities with Deterministic and Stochastic Signal Sources*, Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University, Blacksburg, Virginia (February 10, 2000).