Study of Higher Order Dispersion Effects on Soliton Propagation In Dispersion Shifted Fiber

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ABSTRACT : In this paper I studied higher order dispersion (β_3) effects on soliton propagation in dispersion shifted fiber (DSF) with solving numerically the Nonlinear Schrödinger Equation (NLSE). Initial pulse have hyperbolic-secant shape and power that give a fundamental soliton, fiber loss is compensated by pre-emphasis method. I used Symmetrized Split Step Fourier (SSSF) method to solve the NLSE and compare soliton propagation in DSF from result of NLSE with β_3 is included and β_3 is negligible by measure root mean square (rms) pulse width of soliton that determines the bit rate of soliton communication systems.

KEYWORDS : Higher order dispersion (β_3), soliton propagation, pre-emphasis method, dispersion shifted fiber (DSF), Nonlinear Schrödinger Equation (NLSE), Symmetrized Split Step Fourier (SSSF) method.

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1 INTRODUCTION

The word *soliton* was coined by Krusal and Zabusky in 1965 to describe the particle like properties of pulse envelope in dispersive nonlinear media; under certain condition the envelope not only propagates undistorted but survives collisions just as particles do [1].

In 1973 Hasegawa and Tappert suggested the possibility of soliton propagation in optical fiber in their paper on *Applied Physic. Letter. 23, 142 (1973)* [2]. Soliton in optical fibers were first observed by L.F. Mollenauer *et al*. in an expriments that used a mode-locked color-center laser to obtain short ($T_{FWHM} \approx 7$ ps, *FWHM = Full Width at Half Maximum*) optical pulse near 1.55 µm, a wavelength close to which the fiber loss is minimum [2].

DSF is fiber that zero dispersion wavelength shifted to near 1,55 µm, here fiber loss is minimum. In DSF when operating wavelength is 1.55 µm, GVD (Group Velocity Delay) parameter (β_2) is close to zero so higher order dispersion (β_3) must be considered [1].

The propagation of optical pulse envelope for pulse width ≥ 0.1ps in a nonlinear fiber included higher order dispersion and maintain its polarization is goverend by Nonlinear Shrödinger Equation (NLSE), can be written as [1]

$$\frac{\partial A}{\partial z}+\frac{a}{2}A+\frac{i}{2}b_2\frac{\partial^2 A}{\partial T^2}-\frac{1}{6}b_3\frac{\partial^3 A}{\partial T^3}=ig\,|A|^2 A \tag{1}$$

where :
A = Pulse envelope (slowly varying variable).
g = Nonlinearity parameter.

$T = t - z/vg$, where T = time in a reference frame moving with pulse; t = physical time, z = the direction of pulse propagation along fiber axis and v_g = the group velocity at the center wavelength. Eq. (1) can be written in normalized form as:

$$\frac{\partial U}{\partial x}+\left(L_D.\frac{a}{2}\right)U+\mathrm{sgn}(b_2)\frac{i}{2}\frac{\partial^2 U}{\partial t^2}-\mathrm{sgn}(b_3)\frac{1}{6}b_3\frac{L_D}{L_D^{'}}\frac{\partial^3 U}{\partial t^3}=iN^2|U|^2U \tag{2}$$

where : $t=\dfrac{T}{T_0}$; $L_D=\dfrac{T_0^{\,2}}{|b_2|}$; $L_D^{'}=\dfrac{T_0^{\,3}}{|b_3|}$; $L_N=\dfrac{1}{gP_{avg}}$; $x=\dfrac{z}{L_D}$; $N^2=\dfrac{L_D}{L_N}$; $U=\dfrac{A}{\sqrt{P_0}}$ and

sgn(β_2) (or *sgn*(β_3)) takes values +1 and -1 depending on whether β_2 (or β_3) is positive or negative [1]. U, α P_0, ξ, T_0, L_D, L'_D and L_N is normalized pulse envelope, fiber loss, peak input pulse power, input pulse width, normalized distance, dispersion length, higher order dispersion length and nonlinear length. Soliton exist only for integer values of N, soliton corresponding to N = 1 is called fundamental soliton [1].

2 RMS PULSE WIDTH AND PRE-EMPHASIS METHOD

2.1 RMS Pulse Width

In a dispersive nonlinear fiber, the pulse shape at the fiber output can deviate considerably from the input pulse shape and can have much more complicated forms. RMS pulse width is a proper measure of the pulse width that propagetes in nonlinear fiber. ***RMS pulse width*** defined by [4] :

$$s = \left[< t^2 > - < t >^2 \right]^{\frac{1}{2}} \tag{3}$$

where :

$$< t^n > = \frac{\int_{-\infty}^{\infty} t^n \left| U(z,t) \right|^2 dt}{\int_{-\infty}^{\infty} \left| U(z,t) \right|^2 dt} \tag{4}$$

RMS pulse width in normalized units defined by [4] :

$$s = \left[< t^2 > - < t >^2 \right]^{\frac{1}{2}} \tag{5}$$

where :

$$< t^n > = \frac{\int_{-\infty}^{\infty} t^n \left| U(x,t) \right|^2 dt}{\int_{-\infty}^{\infty} \left| U(x,t) \right|^2 dt} \tag{6}$$

RMS pulse width is of interest since it provides a usefulmetric for assessing performance limitations in fiber-optic communication systems. The RMS pulse width is directly related to the maximum data rate through the commonly used design criterion [1,4] :

$$s_t R_b < \frac{1}{4} \tag{7}$$

where σ_t is *RMS pulse width* at fiber ouput and R_b is *bit rate.*

2.2 Pre-Emphasis Method

A amplifier is placed periodically along the fiber link to compensate fiberl loss and it's gain is adjusted so that fiber loss between two amplifiers exactly compensated.

Fiber loss reduce the effective value of N as the soliton propagates so in pre-emphasis method is N is made greater than 1, where :

$$N = \sqrt{\frac{aL}{1 - \exp(-aL)}} \tag{8}$$

Where L is amplifier spacing and to avoid large fluctuations is necessary to keep :

$$L << \frac{p}{20} L_D \tag{9}$$

3 METHODOLOGY

I use SSSF method to solve numerically Eq. (2), algorithm of SSSF method is written using MatLab.

3.1 SSSF Method

Eq. (2) can be written as [2]

$$\frac{\partial U}{\partial x} = \left(\hat{D} + \hat{N} \right) U \tag{10}$$

$\hat{D}$ and $\hat{N}$ is linear operator and nonlinear operator, where form Eq. (2)

$$\hat{D} = -\operatorname{sgn}(b_2)\frac{i}{2}\frac{\partial^2 u}{\partial t^2} + \operatorname{sgn}(b_3)\frac{1}{6} b_3 \frac{\mathrm{L_D}}{\mathrm{L_D'}}\frac{\partial^3 u}{\partial t^3}\ ;\ \hat{N} = iN^2 |U|^2 .$$

SSSF method nonlinear effects assumed at the middle of step size segment and calculated its effect for the whole segment [3]. The approximation solution of NLSE by SSSF method is :

$$U(x+h,T) = \exp\left(\frac{h}{2}\hat{D}\right)\exp\left(\int_x^{x+h} \hat{N}(x')dx'\right)\exp\left(\frac{h}{2}\hat{D}\right)U(x,T)$$
$$\int_x^{x+h} \hat{N}(x')dx' \approx \frac{h}{2}\left(\hat{N}(x) + \hat{N}(x+h)\right) \tag{11}$$

h is normalized step size parameter. Accuration of SSSF method is third order of h ($O(h^3)$). h chosen so that the maximum phase shift (Φ_{max}) due the nonlinear operator is below certain value as

$$g\,\left|A_p\right|^2 h \le \frac{\Phi_{\max}}{L_D} \tag{12}$$

A_p is peak pulse envelope. It has been reported that when Φ_{max} is below 0.05 rad, SSSF method gives a good result for simulation of most contemporary optical communication systems [4].

3.2 Soliton Propagation

We simulated soltion propagation with hyperbolic-secant input to the NLSE as

$$U(0,\tau) = \sec h(\tau) \tag{13}$$

First, I compare propagation of fundamental soliton in fiber that loss is zero and in fiber that loss is not zero using pre-emphasis mehod using Eq. (13) as input to Eq. (2). Then, I use Eq. (13) as input to Eq. (2) to simulate propagation of fundamental soliton with β_3 is included and β_3 is negligible and compare the result. Finaly using rms pulse width , I compare the bit rate transmission distance product of both.

4 RESULTS

4.1 Pre-Emphasis Method

Fig.1 is propagation of fundamental soliton in *DSF* as result of *SSSFM* using Eq. (13) as input, where $\alpha = 0$ *dB*, β_2 = -1 ps^2/km, β_3 = 0.5 ps^3/km, T_0 = 1 ps and h = 0.01.

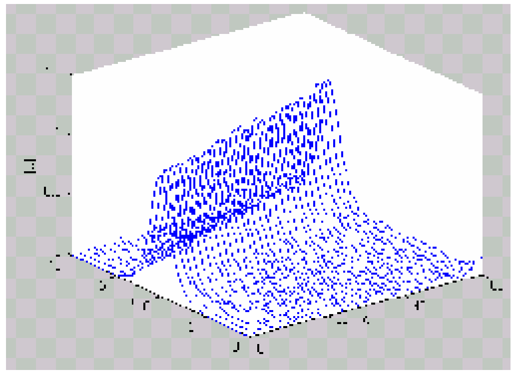


FIGURE 1: Propagation of fundamental soliton, α = 0 *dB*.

Fig.2 is propagation of fundamental soliton in *DSF* as result of *SSSFM* and pre-emphasis method with $L = 0{,}1L_D$, using Eq. (13) as input, where α = 0.2 *dB*, β_2 = -1 ps^2/km, β_3 = 0.5 ps^3/km, T_0 = 1 ps and h = 0.01.

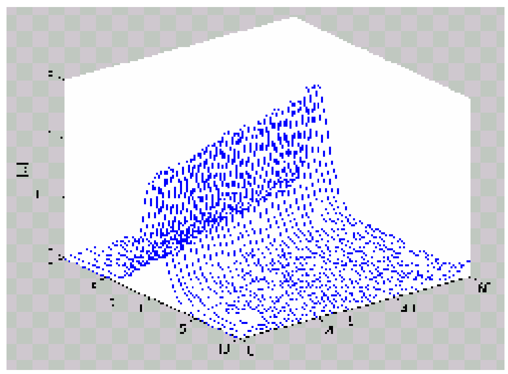
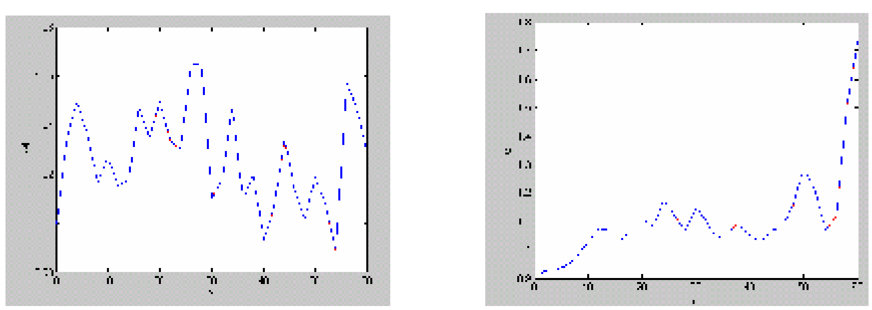
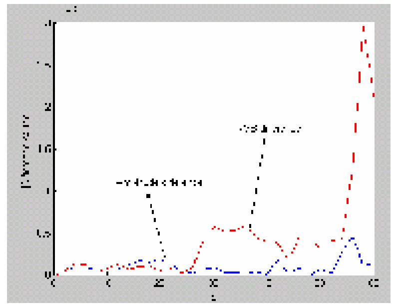


FIGURE 2 : Propagation of fundamental soliton, α = 0.2 *dB*.

Fig. 3(a) and 3(b) is amplitude comparison and RMS pulse width comparison for α = 0 *dB* and α = 0.2 *dB* as function of propagation distance. Fig. 3(c) is comparison of them, using their absolut difference values.



(a) (b)



(c)

FIGURE 3 : (a) Amplitude comparison. (b) RMS pulse width comparison. (c) Amplitude and RMS pulse width difference.

4.2 Higher Order Dispersion Effects

Fig.4 is propagation of fundamental soliton in *DSF* as result of *SSSFM* and pre-emphasis method with $L = 0{,}1L_D$ using Eq. (13) as input with β_3 is negligible ($\beta_3 = 0$), where $\alpha = 0.2$ *dB*, $\beta_2 = -1$ ps^2/km, $T_0 = 1$ ps and $h = 0.01$. While, at Fig.2 we can see propagation of fundamental soliton with β_3 is included.

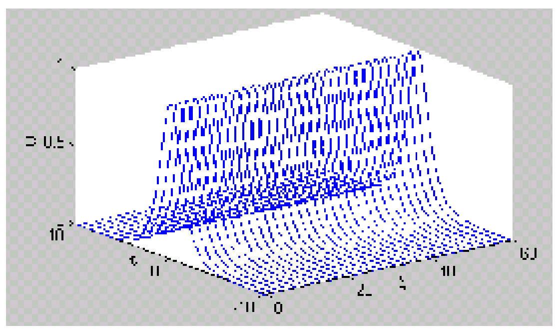
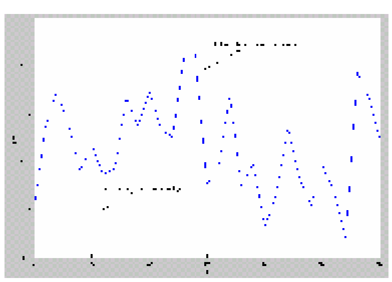
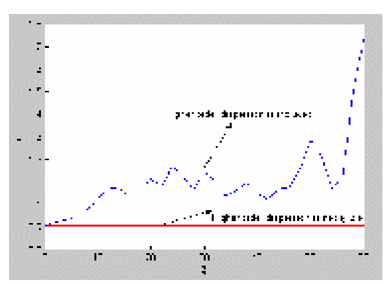


FIGURE 4 : Propagation of fundamental soliton, $\beta_3 = 0$.

Fig.5 (a) and 5 (b) is comparison of amplitude and RMS-pulse width of fundamental soliton propagation in *DSF* as result of with β_3 is negligible ($\beta_3 = 0$ ps^3/km) and $\beta_3 = 0.5$ ps^3/km. Fig. 5(c) is comparison of them, using their absolut difference values [4].



(a)



(b)

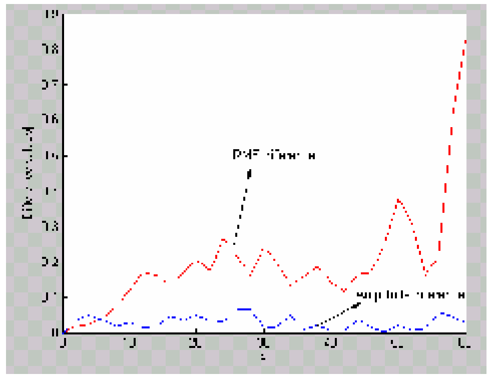


FIGURE 5 : (a) Amplitude comparison. (b) RMS pulse width comparison.
(c) Amplitude and RMS pulse width difference.

5 DISSCUSION

We can compute that average value of amplitude difference between results of SSSFM with $\alpha = 0$ and α = 0.2 dB that compensated by pre-emphasis method along propagation (ξ = 60) is $9.6096.10^{-5}$ and $4.3362.10^{-4}$ for rms pulse width difference. So we can conclude that pre-emphasis method gives a accurate result to compensate fiber loss.

From Fig.(2) and Fig.(4) we can see that effects of higher dispersion change the position of pulse peak so the pulse is not symmetry anymore around $\tau = 0$ and make ripples near leading and trailing edge of the pulses.

Table1 is rms pulse width as function of normalized distance from Fig 3.(a) and 3.(b), for ξ = 0, 10, 20, 30, 40, 50 and 60.

TABLE 1 : RMS pulse width as function of ξ .

ξ (Normalized distance)	σ (for $\beta_3 = 0$ ps^3/km)	σ (for β_3 = 0.5 ps^3/km)
0	0.90690	0.9069
10	0.90700	1.028
20	0.90712	1.1115
30	0.90708	1.1497
40	0.90695	1.0484
50	0.90694	1.2873
60	0.90703	1.7439

From table1 we can see that for fiber optics with $\beta_3 = 0$ so the rms pulse width is almost constant, using Eq. (7) we get constant bit rate of transmission is about, $R_b < 0.2756$ Gb/s . Finaly we use the value of rms pulse width in table1 and compute bit rate transmission distance product, the result is in table2.

TABLE 2 : Bit rate transmisson distance product.

L_T (Km) = $\xi.L_D$ (Distance)	R_bL_T (Gb/s.Km) (for β_3 = 0 ps^3/km)	R_bL_T (Gb/s.Km) (for β_3 = 0.5 ps^3/km)
10	< 2.756	< 2.431907
20	< 5.512	< 4.498426
30	< 8.268	< 6.523441
40	< 11.024	< 9.538344
50	< 13.78	< 9.710246
60	< 16.536	< 8.601411

From table2 we can conclude that effects of higher order dispersion is also reduce the bit rate transmission distance product.

6 CONCLUSION

In his paper, the simulation results show that pre-emphasis method gives a accurate result to compensate fiber loss.

When higher order dispersion is negligible, the pulse shape (peak and rms pulse width) is almost not change along the propagation so give a almost constant bit rate for the system. When higher order dispersion is included so it's effects will increase the rms pulse width of pulse along the propagation which reduce the bit rate transmission distance product.

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