Study of Super-Gaussian Pulse Propagation In Nonlinear Fiber Optics

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***Abstract*— In this paper I study super-Gaussian pulse propagation in nonlinear fiber optics with solving numerically the Nonlinear Schrödinger Equation (NLSE). The initial pulse has pre-chirped super-Gaussian pulse shape and fiber loss is negligible. I use Symmetrized Split Step Fourier (SSSF) method to solve the NLSE then compare super-Gaussian pulse propagation in nonlinear fiber optics and in linear fiber optics by measuring root mean square (rms) pulse width of pulse that determines the bit rate of fiber. From the results of numerical simulation, I found out that nonlinear fiber gives higher bit rates than linear fiber and effect of pre-chirp and edge sharpness parameter of super-Gausian pulse decreases the bit rate both for linear and nonlinear fiber.**

***Keywords*— Super-Gaussian pulse, nonlinear fiber optics, Nonlinear Schrödinger Equation (NLSE), Symmetrized Split Step Fourier (SSSF) method.**

I. INTRODUCTION

The nonlinearities in optical fibers fall into two categories. One is stimulated scattering (Raman and Brillouin), and the other is the optical Kerr effect due to changes in the refractive index with optical power. While stimulated scatterings are responsible or intensity dependent gain or loss, the nonlinear refractive index is responsible for intensity dependent phase shift of the optical signal. One major difference between scattering effects and the Kerr effect is that stimulated scatterings have threshold power levels at which the nonlinear effects manifest themselves while the Kerr effect doesn't have such a threshold [1].

Super-Gaussian pulse is a pulse with steeper leading and trailing edges broadens more rapidly. Pulse emitted by directly modulated semiconductor lasers fall in this category [2].

The propagation of optical pulse envelope for pulse width ≥ 0.1ps in a nonlinear lossless single mode fiber with only Kerr effects is considered is goverened by NLSE, can be written as [3]

$$\frac{\partial A}{\partial z}+\frac{i}{2}b_2\frac{\partial^2 A}{\partial T^2}=ig\,|A|^2A \qquad (1)$$

where :

A = Pulse envelope (slowly varying variable).

g = Nonlinearity parameter.

$T = t - z/v_g$, where T = time in a reference frame moving with pulse; t = physical time, z = the direction of pulse propagation along fiber axis and v_g = the group velocity at the center wavelength.

Eq. (1) can be written in normalized form as:

$$\frac{\partial U}{\partial x}+\mathrm{sgn}(b_2)\frac{i}{2}\frac{\partial^2 U}{\partial t^2}=iN^2|U|^2U \qquad (2)$$

where : $t=\frac{T}{T_0}$; $L_D=\frac{T_0^2}{|b_2|}$; $L_N=\frac{1}{gP_0}$; $x=\frac{z}{L_D}$; $N^2=\frac{L_D}{L_N}$; $U=\frac{A}{\sqrt{P_0}}$ and $sgn(\beta_2)$ takes values +1 and -1 depending on whether β_2 (or β_3) is positive or negative. U, P_0, ξ, T_0, L_D, and L_N are normalized pulse envelope, peak input pulse power, normalized distance, input pulse width, dispersion length, and nonlinear length.

II. METHODOLOGY

I use SSSF method to solve numerically Eq (2), algorithm of SSSF method is written using MatLab.

A. *Symmetrized Split Step Fourier (SSSF) method*

Eq (2) can be written as [3]

$$\frac{\partial U}{\partial x}=\left(\hat{D}+\hat{N}\right)U \qquad (3)$$

$\hat{D}$ and $\hat{N}$ are linear operator and nonlinear operator, where $\hat{D}=-\mathrm{sgn}(b_2)\frac{i}{2}\frac{\partial^2 u}{\partial t^2}$; $\hat{N}=iN^2|U|^2$.

In SSSF method nonlinear effects assumed at the middle of step size segment and calculated its effect for the whole segment. The approximation solution of NLSE by SSSF method is [3]

$$U(x+h,T)=\exp\left(\frac{h}{2}\hat{D}\right)\exp\left(\int_x^{x+h}\hat{N}(x')dx'\right)\exp\left(\frac{h}{2}\hat{D}\right)U(x,T)$$
$$\int_x^{x+h}\hat{N}(x')dx'\approx\frac{h}{2}\left(\hat{N}(x)+\hat{N}(x+h)\right) \qquad (4)$$

h is a step size parameter. Accuration of SSSF method is third order of h ($O(h^3)$).

B. *Accuracy of SSSF Method and rms Pulse Width*

NLSE in Eq (2) can be solved analytically for N = 1, *sgn*(β_2) = -1, and input pulse is $U(0,\tau)$ = *sech*(τ), the solution is known as fundamental soliton where pulse remains unchanged during propagation [2].

Normalized Square Deviation (NSD) parameter is used to measure accuracy of numerical solution to solve NLSE using SSSF method compare to analytic solution. NSD is defined by [1] :

$$NSD = \frac{\int_{-\infty}^{\infty} \left| U_A(x,t) - U_B(x,t) \right|^2 dt}{\int_{-\infty}^{\infty} \left| U(0,t) \right|^2 dt} \qquad (5)$$

where U_A is output field envelop using SSSF Method and U_B is output field envelop using analytic solution.

In a dispersive nonlinear fiber, the pulse shape at the fiber output can deviate considerably from the input pulse shape and can have much more complicated forms. The rms pulse width is a proper measure of the pulse width that propagetes in nonlinear fiber. The rms pulse width is defined by [2] :

$$s = \left[< t^2 > - < t >^2 \right]^{\frac{1}{2}} \qquad (6)$$

where :

$$< t^n > = \frac{\int_{-\infty}^{\infty} t^n \left| U(z,t) \right|^2 dt}{\int_{-\infty}^{\infty} \left| U(z,t) \right|^2 dt} \qquad (7)$$

The *rms pulse width* in normalized units defined by :

$$s = \left[< t^2 > - < t >^2 \right]^{\frac{1}{2}} \qquad (8)$$

where :

$$< t^n > = \frac{\int_{-\infty}^{\infty} t^n \left| U(x,t) \right|^2 dt}{\int_{-\infty}^{\infty} \left| U(x,t) \right|^2 dt} \qquad (9)$$

The rms pulse width is of interest since it provides a useful metric for assessing performance limitations in fiber-optic communication systems. The rms pulse width is directly related to the maximum data rate through the commonly used design criterion [1,2,3] :

$$s_t R_b < \frac{1}{4} \qquad (10)$$

where σ_t is rms pulse width at fiber ouput and R_b is bit rate.

C. *Super-Gaussian Pulse Propagation*

Normalized super-Gaussian pulse is described by :

$$U(0,\tau) = \exp\left[\frac{-1+iC}{2} t^{2m} \right] \qquad (11)$$

where C is a chirp parameter and parameter m controls the degree of edge sharpness [2]. An optical pulse is said to be chirped if its carrier frequency changes with time in a deterministic fashion, if there are random variations in the carrier frequency, e.g. phase noise, this is generally not referred to as chirp [4]. For m = 1, the pulse become Gaussian pulse. Fig. 1 shows super-Gaussian pulse shape for m = 2, 3, 5 and 10 where for larger value of m, the pulse becomes square with sharper leading and trailing edges.

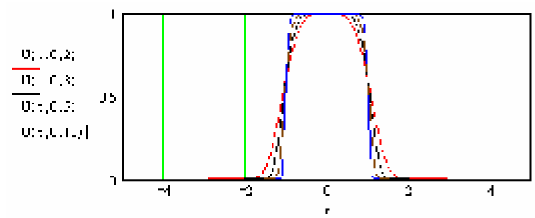

Fig. 1. Super Gaussian pulse shape for m = 2, 3, 5 and 10.

The rms pulse width of super-Gaussian pulse along propagation in linear fiber is given by [3] :

$$\frac{s(x)}{s(0)} = \sqrt{1 + \mathrm{sgn}(b_2) \frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} Cx + \frac{\Gamma\left(2 - \frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \left(1 + C^2\right) m^2 x^2} \qquad (12)$$

While for in nonlinear fiber is found by using Eq (11) as input to Eq (2) and solve it using NLSE method, then I compare the results.

III. RESULTS

A. *Accuracy of SSSF Method*

The NSD between SSSF method and analytic solution for fundamental soliton along propagation in nonlinear fiber from $\xi = 0$ until $\xi = 30$ is shown in Fig. 2.

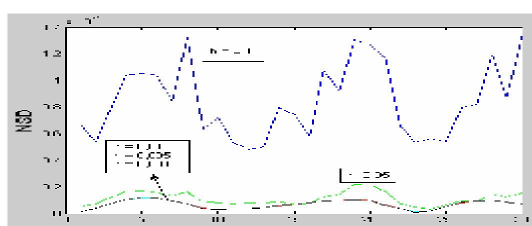

Fig. 2. NSD between SSSF method and analytic solution.

The average value for NSD along $\xi = 0$ until $\xi = 30$ for different h is shown in Table I.

TABLE I
AVERAGE VALUE OF NSD

h	Average value of NSD
0.100	8,45631.10^{-8}
0.050	1,1197.10^{-8}
0.010	6,6839.10^{-9}
0.005	6,6711.10^{-9}
0.001	6.6649.10^{-9}

B. *Super-Gaussian Pulse Propagation*

For $m = 2$, $C = 0$, $sgn(\beta_2) = -1$ comparison of rms pulse width (normalized to input rms pulse width) along propagation in nonlinear fiber (for $N = 1, 2, 3$ and 4) and in linear fiber from $\xi = 0$ until $\xi = 100$ is shown in Fig.3.

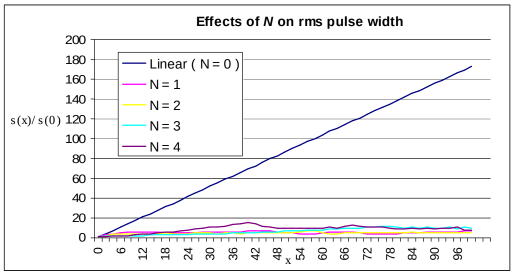

Fig. 3. rms pulse width in linear and nonlinear fiber, $N = 1, 2, 3$ and 4.

Fig. 4. shows effects of m on rms pulse width (normalized to input rms pulse width) along propagation in linear fiber for $C = 0$, and $sgn(\beta_2) = -1$. While Fig. 5. shows for nonlinear fiber where $N = 1$, both from $\xi = 0$ until $\xi = 100$.

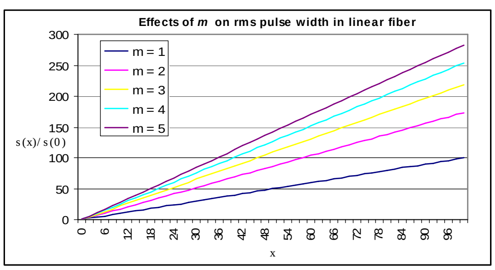

Fig. 4. rms pulse width in linear fiber, $m = 1, 2, 3, 4$ and 5.

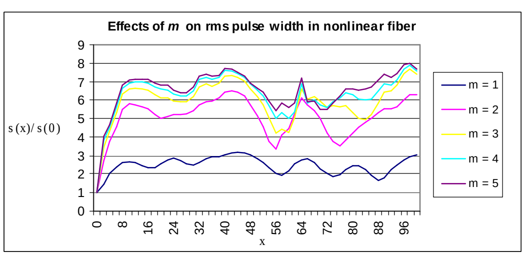

Fig. 5. rms pulse width in nonlinear fiber ($N = 1$), $m = 1, 2, 3, 4$ and 5.

Fig. 6. shows effects of pre-chirping, C, on rms pulse width (normalized to input rms pulse width) along propagation in linear fiber for $m = 2$ and $sgn(\beta_2) = -1$. While Fig. 7. shows for nonlinear fiber where $N = 1$, both from $\xi = 0$ until $\xi = 100$.

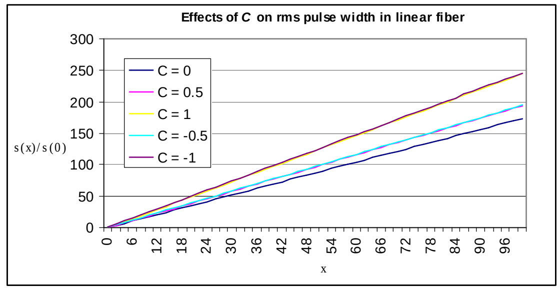

Fig. 6. rms pulse width in linear fiber, $C = 0, 0.5, 1, -0,5$ and -1.

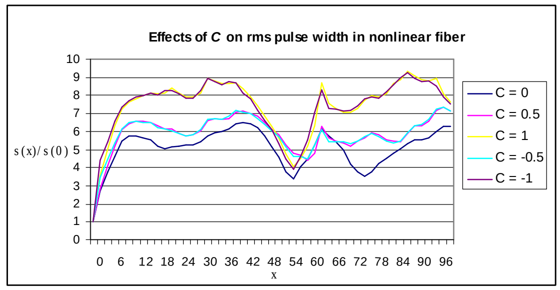

Fig. 7. rms pulse width in nonlinear fiber (N =1), $C = 0, 0.5, 1, -0,5$ and -1.

IV. DISCUSSION

From Fig. 2., we can see that NSD value becomes smaller that mean SSSF method more accurate for smaller h. Table I shows that SSSF method to solve NLSE (Eq (2)) gives accurate result which NSD is very small value in order -8 to -9.

From Fig. 3., we can see that in linear fiber, the rms pulse width increases monotonically but fluctuates in nonlinear fiber, where N gives a small change in rms pulse width fluctuation Nonlinear fiber also reduces the increasing of rms pulse width compare to linear fiber.

From Fig. 4, and Fig. 5, we can see that both in linear and nonlinear fiber, for larger value of m, then rms pulse width increases with a faster rate. We can see also in Fig. 4, and Fig. 5, that in linear fiber, the rms pulse width increases monotonically but fluctuates in nonlinear fiber for every m.

TABLE II
BIT RATE AS A FUNCTION OF m

m	Linear fiber		Nonlinear fiber	
	σ_t(ps) at $\xi = 100$	R_b (Gb/s)	σ_t(ps) at $\xi = 100$	R_b (Gb/s)
1	707.14	< 0.35	21.49	< 11.64
2	1007.03	< 0.25	36.41	< 6.87
3	1233.28	< 0.20	41.83	< 5.98
4	1422.92	< 0.18	42.56	< 5.87
5	1589.88	< 0.16	42.95	< 5.82

Using Eq. (2) we can calculate bit rate, R_b, for distance, ξ = 100 as a function of m in linear and nonlinear fiber. Table II shows the results if we use T_0 = 10 ps. From Table II we can see that the bit rate of nonlinear fiber is about thirty times greater than the bit rate of linear fiber and for both the bit rate decreases for m increases.

From Fig. 6, and Fig. 7, we can see that both in linear and nonlinear fiber, for larger value of C (positive or negative) then rms pulse width increases with a faster rate. Positive and negative C for the same value, both give almost the same effects on rms pulse width. We can see also in Fig . 6 and Fig. 7, that in linear fiber, the rms pulse width increases monotonically but fluctuates in nonlinear fiber for every C.

TABLE III
BIT RATE AS A FUNCTION OF C

C	Linear fiber		Nonlinear fiber	
	σ_t (ps) at ξ = 100	R_b (Gb/s)	σ_t (ps) at ξ = 100	R_b (Gb/s)
0	1007.03	< 0.25	36.41	< 6.87
0.5	1123.67	< 0.22	41.40	< 6.04
1	1420.63	< 0.18	44.19	< 5.66
-0.5	1128.11	< 0.22	41.39	< 6.04
-1	1427.65	< 0.18	43.58	< 5.74

Using Eq. (2) we can calculate bit rate, R_b, for distance, ξ = 100 as a function of C for linear and nonlinear fiber. Table III shows the results if we use T_0 = 10 ps. With there is a pre-chirping effect, we can see that the bit rate of nonlinear fiber is still about thirty times greater than the bit rate of linear fiber and for both the bit rate decreases for C increases (postive or negative). We can also see from Table III that for positive and negative C for the same value, both give almost the same bit rate for linear and nonlinear fiber.

V. Conclusion

The SSSF gives accurate result to solve NLSE which NSD is very small value in order -8 to -9 for step size 0.1 until 0.001.

In linear fiber, the rms pulse width increases monotonically but fluctuates in nonlinear fiber, where nonlinear fiber reduces the increasing of rms pulse width compare to linear fiber.

For larger value of m parameter of super-Gaussian pulse then rms pulse width increases with a faster rate and decreases the bit rate of fiber.

For larger value of pre-chirping, C (positive or negative) then rms pulse width increases with a faster rate and decreases the bit rate of fiber. Positive and negative C for the same value, both give almost the same effects on rms pulse width. The bit rate of nonlinear fiber is about thirty times greater than the bit rate of linear fiber.

References

[1] Jong-Hyung Lee, *Analysis and Characterization of Fiber Nonlinearities with Deterministic and Stochastic Signal Sources*, Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University, Blacksburg, Virginia (February 10, 2000).

[2] Agrawal, P. Govind, *Fiber-Optic Communication Systems*, Willey Series In Microwave And Optical Engineering, (1992).

[3] Agrawal, P. Govind, *Nonlinear Fiber Optics*, Academic Press, Sandiego, LA, (1989).

[4] Natee Wongsangpaiboon, *Variational Calculation of Optimum Dispersion Compensation For Non-Linear Dispersive Fibers*, Thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University, Blacksburg, Virginia (May 17, 2000).